# Transient Airfoil Aerodynamics of Vertical Axis Wind Turbines

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## Abstract

This paper presents a new method to predict the performance of vertical axis wind turbines with airfoil data and relative angles of attack. This new technique offers the ability to investigate the proportional effects on performance of airfoil design, rigging angle, solidity and tip-speed-ratio. This method offers valuable insights into the aerodynamics of VAWT operation, as well as providing an efficient design tool to compare various airfoil cross sections for wind turbine power generation. The new technique presented in this paper can be used to develop more efficient VAWT designs with higher power output and more effective operation in a variety of wind conditions and installation locations.

**Keywords**: Aerodynamics, VAWT, airfoil

## 1. Introduction

Vertical axis wind turbines (VAWTs) can be placed in a variety of locations as they offer omnidirectional operation. A high number of VAWT designs and geometric configurations have been developed each with different operating parameters and aerodynamic principles for operation. Typically, the more effective VAWT designs utilize two or three vertical airfoil blades [1]. The lift type VAWTs can be further categorized into straight blade (H-rotor) or curved blade (Darrius) designs.

During the operation of the lift type VAWTs, complex flow conditions are developed that are highly variable throughout the rotor rotation. Complex flow conditions arise during operation with dynamic lift and drag forces, coupled with airfoil stalling are generated during every rotation. Previous studies have shown that the inverse relationship between the solidity of an H-rotor VAWT and its optimum tip-speed-ratio for maximum power output [2]. A design with a high solidity will be unable to achieve a high tip-speed-ratio, which limits the power production. Conversely, a low solidity has a lower maximum power output due to the smaller surface area of the airfoils which limits the available thrust to rotate the turbine. Balancing these effects is critical to achieving high power output. Beri and Yao [2] performed a comprehensive analysis of various solidities and tip-speed-ratios, with experimental methods, and determined the optimal solidity to be between 0.2 and 0.4, which performed optimally at tip-speed-ratios of 3.5 and 4.5, respectively. However, a turbine with solidity of 0.4 has smaller range of tip-speed-ratios with positive torque generation, which makes it difficult to achieve optimal tip-speed-ratios.

Each airfoil design had its own characteristic aerodynamic performance, whereby lift and drag coefficients can be measured for various angles of attack to determine the performance of the airfoil design. In this paper, a method is developed that utilizes the aerodynamic performance data of an airfoil to predict the transient performance of a VAWT. The technique can be applied to a variety of geometrical and operational conditions, such as airfoil design, rigging angles, solidity and tip-speed-ratios, providing valuable insights into the effects of VAWT geometry and wind conditions on the systems energy generation capabilities. Furthermore, the relative wind speed and relative angle of attack on a VAWT is highly variable throughout the rotor rotation, as well as being highly dependent on tip-speed-ratio (TSR). These non-linear relative proportions, the variable nature of wind, and different operational requirements cause appreciable complications in selecting the optimal airfoil cross-section for a VAWT design. Also, the complex behavior of dynamic stalling adds further complexity to the selection criteria.

A variety of predictive techniques have been developed to predict the performance of a VAWT. The most common technique, the multiple streamtube method, involves defining a thrust coefficient which must be estimated to obtain a solution [3]. This technique has provided good agreement with numerical simulations [4], and experimental data in terms of power output [3], however, to determine the thrust coefficient requires a precise measurement of the downstream wind velocity, which is typically estimated through analytical techniques, detracting from the accuracy of the technique, as well as limiting its ability to compare proportional changes to the system design and operation, limiting its effectiveness as a design tool.

The ambition to provide effective VAWT designs for the wide variety of wind conditions and operating requirements of various uses and locations as produced a vast array of VAWT designs. However, without an effective design tool, the majority of current VAWT designs have been developed with an appreciable amount of guess work, and trial and error methods (typically with experimental or computational fluid dynamics (CFD)). Furthermore, the complex flow conditions in a VAWT design do not lend themselves well to convenient or accurate CFD predications [5, 6]. These resource intensive methods typical do not converge on the best design within the scheduling and financial limits. In this paper, the aerodynamic performance of a wind turbine is investigated based on the lift and drag characteristics of the airfoil to provide a method to more detailed understand of VAWT aerodynamics, as well as providing an efficient technique to design H-rotor VAWTs for optimal aerodynamic performance.

## 2. Formulation of Transient Forces on a VAWT airfoil

This section develops a predictive technique that utilizes lift and drag data of airfoil aerodynamics to predict the performance of an H-rotor VAWT. As illustrated in Fig. 1, the relative angle of attack ($\boldsymbol{\alpha}$), at all rotor positions ($\theta$), can be calculated by

$$\alpha = RA + \frac{\pi}{2}\frac{V}{\omega R}cos\theta \qquad (1)$$

where *RA*, *V*, *R* and $\boldsymbol{\omega}$ represent the rigging angle of the airfoil, freestream air velocity, turbine radius and rotor angular velocity, respectively. The tip-speed-ratio ($\boldsymbol{\lambda}$) is defined as $\frac{\boldsymbol{\omega R}}{\boldsymbol{V}}$, reducing Eq. (1) to

$$\alpha = RA + \frac{\pi}{2\lambda}cos\theta \qquad (2)$$

The lift ($\boldsymbol{C_L}$) and drag ($\boldsymbol{C_D}$) coefficients are defined by

$$C_L = \frac{L}{0.5\rho c V^2} \qquad (3)$$

$$C_D = \frac{D}{0.5\rho c V^2} \tag{4}$$

where $L$, $D$, $\boldsymbol{\rho}$ and $\boldsymbol{c}$ represent the lift and drag forces, air density, and airfoil chord length, respectively. The tangential lift ($\boldsymbol{L_T}$) and drag ($\boldsymbol{D_T}$) forces can be determined by

$$L_T = 0.5 C_L \rho c V^2 \sin\left(\frac{\pi}{4} - RA\right) \tag{5}$$

$$D_T = 0.5 C_D \rho c V^2 \cos\left(RA\right) \tag{6}$$

where $\left[\mathbf{sin}\left(\frac{\boldsymbol{\pi}}{\mathbf{4}} - \boldsymbol{RA}\right)\right]$ and $[\mathbf{cos}\,(\boldsymbol{RA})]$ represent the angle between $F_T$ - L and $F_T$ - D, respectively. In Eqs. (5) and (6) the lift and drag coefficients can be selected based on tabulated or correlated data for specific airfoil geometry, based on the relative angle of attack during rotation (in this paper they are estimated based on extrapolated airfoil data for different angles of attack).

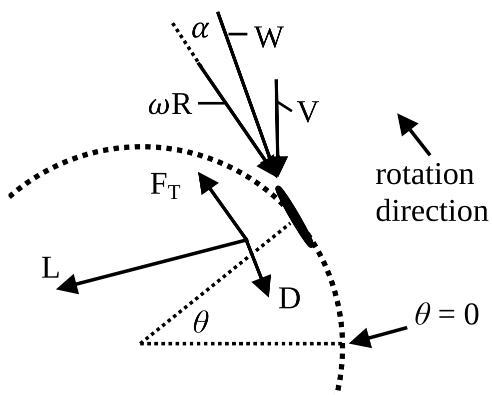


Figure 1: VAWT variables

By adopting the assumption of no blade interaction, the tangential lift and drag forces can be combined to predict the total tangential force ($\boldsymbol{F_T}$) on the rotor blade

$$F_T = L_T - D_T \tag{7}$$

The torque coefficient can be calculated by

$$C_T = \frac{F_T R}{0.5\rho R c V^2} \tag{8}$$

where $R$ represents the turbine radius. The power coefficient can be predicted by the angular velocity and the applied moment on the rotor blade ($\boldsymbol{F_T R}$),

$$C_P = \frac{N F_T R \omega}{0.5\rho A_{ref} V^3} \tag{9}$$

where $N$ represents the blade number, $\boldsymbol{\omega}$ is calculated by $\boldsymbol{\omega} = \frac{\boldsymbol{\lambda V}}{\boldsymbol{R}}$, and the reference area ($\boldsymbol{A_{ref}}$) is calculated by the turbine diameter, reducing Eq. (9) to

$$C_P = \frac{NF_T\lambda}{\rho R V^2} \tag{10}$$

This method provides a new analytical formulation to predict the performance of VAWTs based on the VAWT geometry, airfoil design, and operating conditions.

## 3. Results and Discussion

In this section, the new predictive formulation is applied to a 2-D VAWT with a Clark-Y(B) airfoil to represent a 2D-section of straight blade (H-rotor) VAWT. As presented in Table 1, the formulation is applied to a VAWT with various ranges of blockage ratio, solidity, tip-speed-ratio, and rigging angle to compare the turbine's performance with various turbine geometries and operating conditions.

Table 1: Dimensionless variables of predicted rotational force

| Name | Symbol | Range |
|---|---|---|
| Blockage ratio | *R/c* | 4 to 8 |
| Solidity | $\sigma$ | 0.11 to 0.33 |
| Tip-speed-ratio | *TSR* | 3 to 4 |
| Rigging angle | *RA* | 0 to $\pi/6$ |

The airfoil's lift and drag performance is extrapolated from experimental data [7] of a Clark-Y(B) airfoil, a design for low Reynolds number operation. A piecewise function is adopted for the extrapolated lift correlation,

$$C_L = \begin{cases} 0.0035\alpha^2 + 0.1053\alpha + 0.3354, & x < 0 \\ -0.0035\alpha^{1.9} + 0.1053\alpha + 0.3354, & x \geq 0 \end{cases} \tag{11}$$

The drag coefficient is represented by a quadratic function,

$$C_D = 0.0003\alpha^2 - 9 \times 10^{-5}\,\alpha + 0.015 \tag{12}$$

The extrapolated correlations agree reasonably well with experimental data, which for the purposes of this paper, provide a convenient method to compare various design parameters and operating conditions to investigate the aerodynamic performance of VAWTs. As illustrated in Fig. 2, the experimental correlations provide good agreement in the range of $\theta = -25°\ to\ 35°$ [8]. Beyond this range, the predicted lift and drag magnitudes are increasingly inaccurate. Beyond stall angle of the airfoil the new correlation over predicts the lift force, compared to experimental data at static conditions. However, transient changes in angles of attack typically will allow higher angles to be achieved before flow separation (and stall) is experienced [9].

By adopting the assumption of no blade interaction, the ratio of airfoil chord length to turbine radius (*c/R*) must be limited to maintain accurate predictions (a large *c/R ratio* implies excessive blade interaction). The high blockage caused by the leading airfoil will disrupt the flow conditions for the following airfoil, limiting its lift generation.

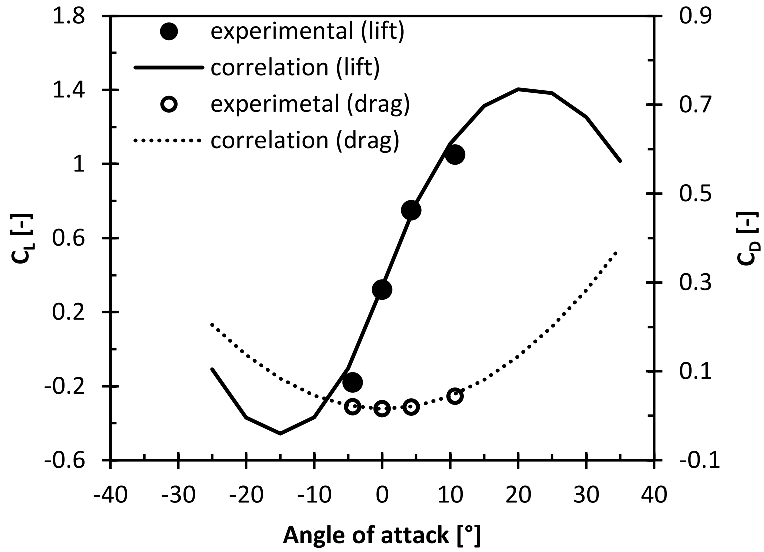


Figure 2: Lift and drag performance of a Clark-Y (B) airfoil

As illustrated in Fig. 3, the generated torque for 1-blade and 2-blade designs follow similar trends, implying the power output from each airfoil follows similar trends in torque generation. This can increase fatigue stress by imposing cyclic stresses on the turbine, limiting its life-span. However, the single blade design has a negative torque value at 270°, which could impose significant problems for a 1-blade design by causing negative thrust during operation. The following analysis is performed on a 2-blade design, as it is a typically the most efficient design [1], and also limits the blade interaction compared to a higher number of airfoils by offsetting the rotor blades 180°.

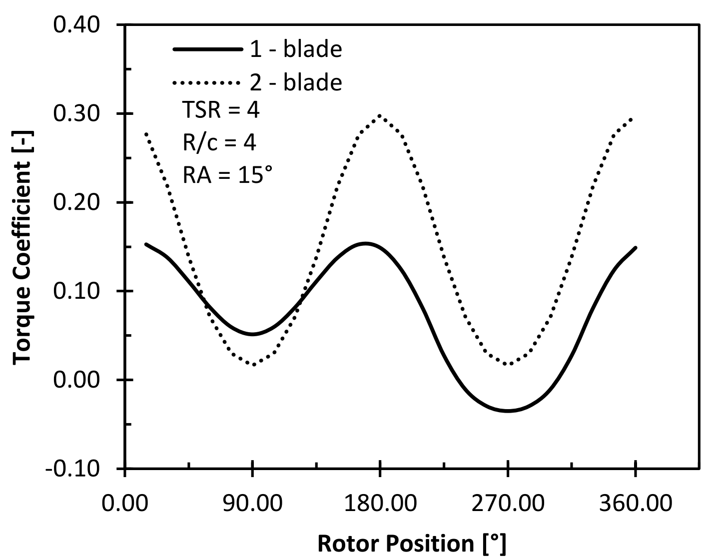


Figure 3: Predicted Torque on a VAWT at all angular position of rotor rotation

In Fig. 4, the predicted power coefficient is illustrated for different solidity ratios and rigging angles. The *r/c* value of 3 corresponds to a solidity of 0.33, which will likely be unable to reach the high tip-speed-ratios needed for optimal power production [9]. However, as illustrated in Fig. 4, increasing the solidity has the potential to significantly increase the theoretical maximum power output of an H-rotor VAWT, if adequate tip-speed-ratios are achieved. Another factor limiting the accuracy of the high *r/c* value is the increased blockage experienced by the trailing airfoil, which will be exposed to more turbulent wind conditions, reducing its lift capabilities.

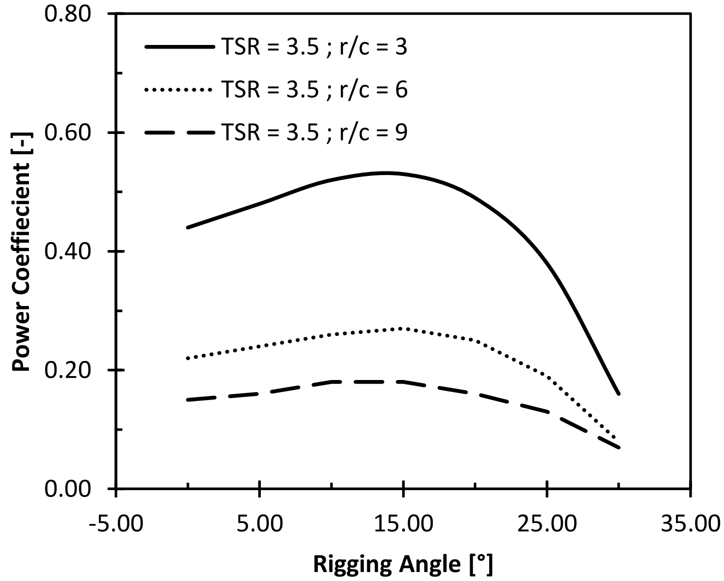


Figure 4: Predicted performance of a VAWT with various blockage ratios

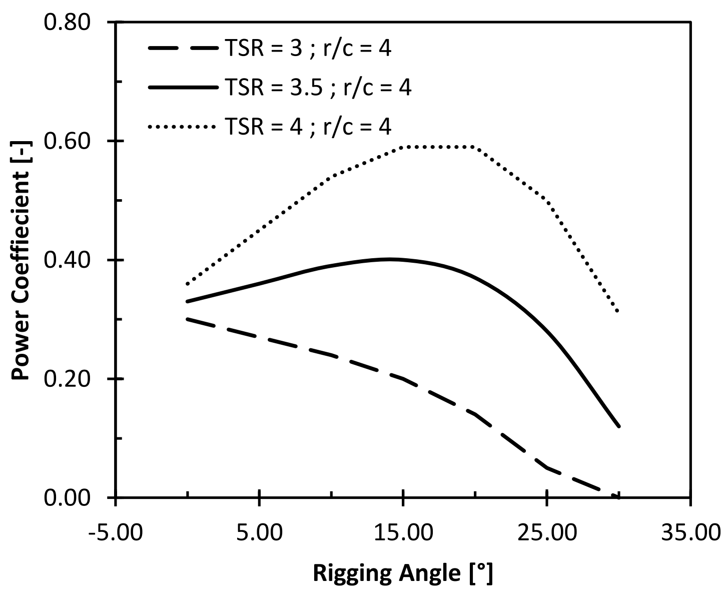


Figure 5: Predicted performance of a VAWT with various tip-speed-ratios

As illustrated in Fig. 5, the tip-speed-ratio has a significant effect on the performance, with similar power outputs at low rigging angles, but with significantly higher optimal power outputs corresponding to higher tip-speed-ratios. A change in tip-speed-ratio from 4 to 3 has the potential to severely limit the performance of the turbine, suggesting that precise active controls are important for optimal performance and supports maintaining solidity closer to 0.2, instead of 0.4. This result also highlights the importance of designing a turbine for the wind conditions of its installations site, as small changes to the operating conditions can drastically limit the power output.

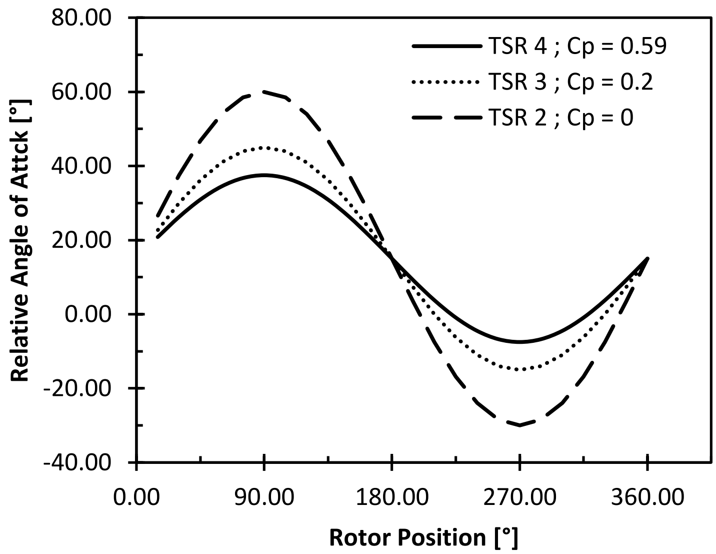


Figure 6: Transient angle of attack at various tip-speed-ratios at all rotor positions

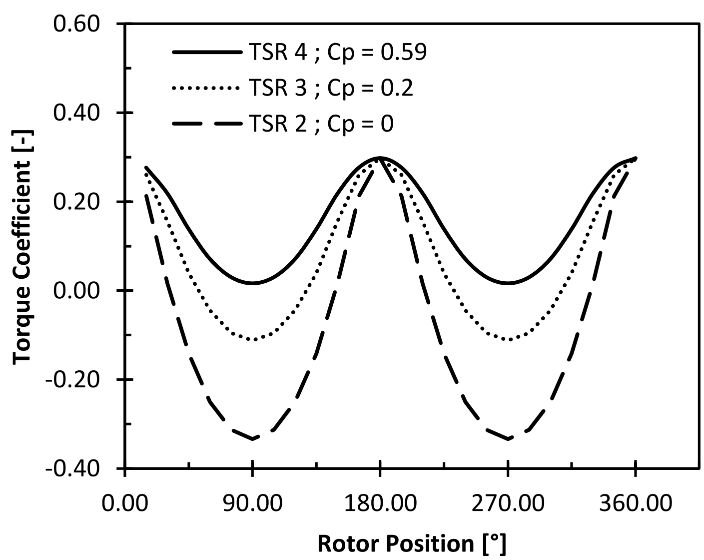


Figure 7: Torque coefficient of various tip-speed-ratios at all rotor positions

The relative angle of attack represents the imposed angle of attack on the airfoil, considering its rigging angle and the relative wind direction, at all rotor positions. As illustrated in Fig. 6, the higher tip-speed-ratios provide a relative angle of attack with lower amplitude, which maintains an angle of attack that is more capable of generating lift, during a greater range of the airfoils rotation around the central axis. Although the greater rotational speed will also cause a higher power output, the predicted change will be proportional, as predicted by Eq. (9). Lowering the tip-speed-ratio from 4 to 3 will reduce the power output caused by the lower rotational speed by one-quarter, with the rest of the predicted power loss being attributed to the changes in the relative angle of attack throughout the rotor rotation. As illustrated in Fig. 7, the effect of the high (in magnitude) relative angles of attack caused by the low tip-speed-ratios causes large negative values in power production at 90° and 270°.

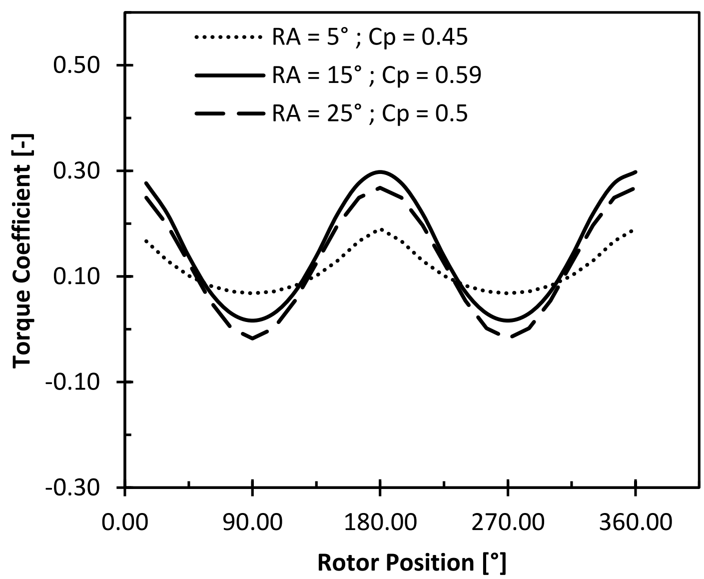


Figure 8: Torque coefficient of various rigging angles at all rotor positions

As illustrated in Fig. 8, altering the rigging angles has an appreciable effect on the trends of torque coefficient, but less impact on the total power output, from 5° to 15°. This figure shows a relatively linear relationship between the change in rigging angle and power output. However, this is only in this range of rigging angles, as they provide similar trends in the relative angles of attack (Fig. 9), with similar ranges of rotation in with power producing range of relative angles of attack. Outside this range of rigging angles, the relationship between the rigging angle and power output will be increasingly non-linear.

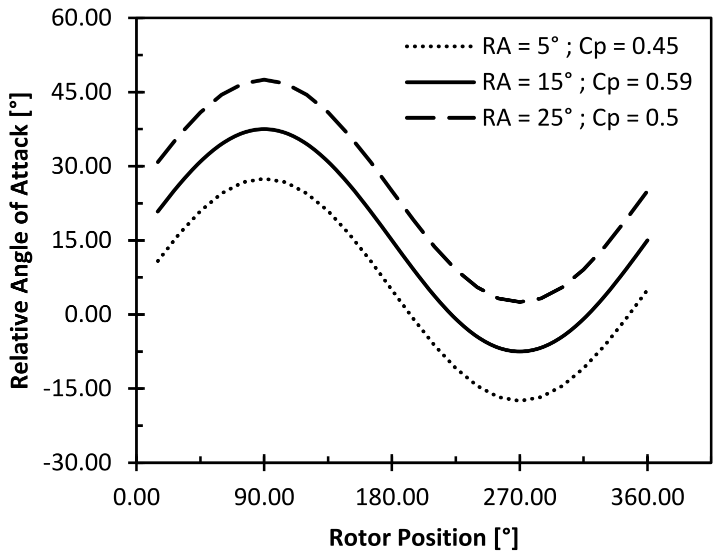


Figure 9: Transient angle of attack at various rigging angles at all rotor positions

## 4. Conclusions

In this paper, a new formulation was developed to predict the performance of a VAWT with aerodynamic data of the airfoils. The predictive technique considers the relative angle of attack at all rotor positions during blade rotation, as well as aerodynamic data on the airfoil to provide predictions in terms of the solidity, chord length, tip-speed-ratio, and rigging angle of the turbines airfoil. The new predictive technique can be used for VAWT design and analysis to provide turbines with improved aerodynamic performance and more effective carbon dioxide mitigation.

## 5. References

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## 6. Acknowledgements

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