Heat Transfer Effectiveness of Borehole Heat Exchangers for Various Grouts: Analysis based on Numerically Simulated Thermal Response Tests

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Abstract

The results are described of thermal response tests (TRTs) based on numerical simulation of different borehole heat exchangers (BHEs), aimed at understanding and predicting the heat transfer effectiveness of borehole heat exchangers for various types of grout. The numerical simulations consider three BHE construction types: coaxial, single U-tube and double U-tube. The results assess quantitatively and qualitatively the influence of grout on thermal efficiency and, further, determine the reasonableness of incurring higher costs in order to improve BHE performance by utilizing grouts with high thermal conductivity.

Keywords: geoenergetics, borehole heat exchanger, thermal response test, grout

Résumé

Cet article présente des résultats d'essais de réponse thermique effectués par simulation numérique de divers échangeurs de chaleur en trou de forage. Le but est de comprendre et de prévoir l'efficacité du transfert de chaleur des échangeurs pour divers types de coulis. Des simulations numériques ont été effectuées pour trois types de construction d'échangeurs thermiques en trou de forage: coaxial, tube en U simple et tube en U double. Les résultats permettent d'évaluer quantitativement et qualitativement l'effet du coulis sur le rendement thermique et, par la suite, de déterminer s'il est raisonnable de dépenser davantage pour des coulis à plus haute conductivité thermique dans le but d'améliorer le rendement des échangeurs de chaleur en trou de forage.

Mots-clés: géothermie, échangeur de chaleur en trou de forage, essai de réponse thermique, coulis

1. Introduction

The borehole heat exchanger (BHE) is becoming an increasingly interesting technology for managing heat in buildings. The local characteristics of the lithosphere and the layers drilled through in creating a borehole well are the geological parameters of a borehole heat exchanger. These characteristics, which influence its energetic effectiveness, are as follows [1]:

a) geothermal gradient,
b) natural earth heat flux,
c) thermal conductivity of rocks,
d) anisotropy of orogenic belt thermal conductivity,

e) rock density,
f) layer thermal capacity,
g) porosity and saturation of layers,
h) type of deposit medium filling the pore and fracture space, and
i) hydrodynamic characteristics of layers and the natural speed of deposit medium filtration.

The energy flux reaching the earth's surface from operating the borehole heat exchanger depends, to a large extent, on the characteristics of its construction. The construction of a borehole heat exchanger involves equipment made for both reaching the original destination of the well and heat exploitation. The construction characteristics are partly determined during the design and implementation process of a borehole heat exchanger, and include the following [1]:

a) depth of insulation cork (or packer),
b) internal well diameter,
c) length of insulation casing,
d) internal and external diameters of insulation casing,
e) drill construction, including number, length and diameter of the casing, quality of material insulating the casing as well as the condition of this material,
f) heat resistance of material of the internal column,
g) course of drill axis,
h) centricity of internal column, and
i) distance between borehole heat exchangers (when more than one is present).

The exploitation parameters depend on the above mentioned geological and construction characteristics. They play a decisive role in the profitability of using drills to create borehole heat exchangers. The exploitation parameters are as follow [1]:

a) average annual heat production,
b) maximum instantaneous heating power,
c) long-term heating power,
d) flux of heat carrier volume,
e) resistance of heating medium flow,
f) time of ground temperature restoration (period of the energy-resource renewability process),
g) temperature of flowing heat carrier,
h) temperature of compressed heat carrier (a function of carrier cooling in the receiving installation),
i) type of heat carrier,
j) time and circularity of exploitation,
k) distance of heat consumer from well,
l) type of heat consumer, working time and level of heat consumption, and
m) local climatic conditions.

2. Thermal response test

In many countries there has been a significant increase in recent years in heating, and heating and cooling, based on heat pumps and BHEs. The most accurate method of determining the thermal properties of borehole heat exchangers are thermal response tests (TRT) [2]. They are carried out on the first hole, usually for large installations with capacities above 100 kW. The thermal performance test results for a borehole heat exchanger can determined the total number of boreholes needed to meet the demand for heat and/or cold.

The test methodology is based on the thermal reaction based on the partial differential equation describing the dynamic relationship $T = T(r, t)$ of the radius *r* range from the borehole heat exchanger, and the duration of the test *t*. This equation has the following form:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{\rho C_p}{\lambda}\frac{\partial T}{\partial t}, \quad (1)$$

which mathematically corresponds to the Theis equation in hydrogeology, but instead examines the distribution of the temperature $T = T(r, t)$ rather than the pressure distribution $p = p(r, t)$. One way to solve this type of partial differential equation is the method of substitution, which reduces the partial differential in equation (1) to an ordinary differential equation. Here, we let the following terms be adopted in substitution [3]:

$$u = \frac{r^2 \rho C_p}{4t\lambda} \quad (2)$$

and

$$\rho C_p = \frac{\lambda}{\alpha}. \quad (3)$$

With the substitution

$$u = \frac{r^2}{4\alpha t} \quad (4)$$

calculating the appropriate partial derivatives of the function $T = T(r, t)$, appearing in equation (1), then the function $u = u(r, t)$ defined by equation (4) is replaced by the equality:

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial r} = \frac{r}{2\alpha t}\frac{\partial T}{\partial u}, \quad (5)$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{\partial^2 T}{\partial u^2}\frac{\partial u}{\partial r}\frac{r}{2\alpha t} + \frac{\partial T}{\partial u}\frac{1}{2\alpha t} = \frac{r^2}{4\alpha^2 t^2}\frac{\partial^2 T}{\partial u^2} + \frac{1}{2\alpha t}\frac{\partial T}{\partial u}, \quad (6)$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial t} = -\frac{r^2}{4\alpha t^2}\frac{\partial T}{\partial u}. \quad (7)$$

Inserting the above into equation (1), which now has the form:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad (8)$$

leads to the relationship:

$$\frac{r^2}{4\alpha^2 t^2}\frac{\partial^2 T}{\partial u^2} + \frac{1}{2\alpha t}\frac{\partial T}{\partial u} + \frac{1}{r}\frac{r}{2\alpha t}\frac{\partial T}{\partial u} = \frac{1}{\alpha}\frac{-r^2}{4\alpha t^2}\frac{\partial T}{\partial u}, \quad (9)$$

whence

$$\frac{r^2}{4\alpha^2 t^2}\frac{\partial^2 T}{\partial u^2} + \frac{1}{\alpha t}\frac{\partial T}{\partial u} + \frac{r^2}{4\alpha^2 t^2}\frac{\partial T}{\partial u} = 0. \quad (10)$$

After the appropriate transformation using equation (4), equation (10) takes the form:

$$u\frac{\partial^2 T}{\partial u^2} + (1+u)\frac{\partial T}{\partial u} = 0. \qquad (11)$$

It can be seen that the applied substitution reduces the partial differential equation (1) to an ordinary differential equation of second order as:

$$u\frac{d^2 T}{du^2} + (1+u)\frac{dT}{du} = 0. \qquad (12)$$

This equation can be reduced to a first-order equation with separation of variables using the substitution [3]

$$\frac{dT}{du} = y, \qquad (13)$$

where $y = y(u)$ and

$$\frac{d^2 T}{du^2} = \frac{dy}{du}. \qquad (14)$$

After substitution, equation (12) takes the form:

$$u\frac{dy}{du} + (1+u)y = 0. \qquad (15)$$

This is a homogeneous linear differential equation which, after separation of variables, can be written as

$$\frac{1}{y}dy = -\frac{1+u}{u}du, \qquad (16)$$

and thus is obtained according to

$$\int \frac{1}{y}dy = -\int \left(\frac{1}{u} + 1\right) du, \qquad (17)$$

or

$$ln|y| = -ln|u| - u + C, \qquad (18)$$

where C is a constant. Therefore

$$y = e^{-ln|u|-u+C}, \qquad (19)$$

and

$$y = \pm e^{C} \cdot \frac{e^{-u}}{u}. \qquad (20)$$

Let $A = \pm e^{C}$, where $A \neq 0$. Then one can solve the equality

$$y = A \cdot \frac{e^{-u}}{u}, \qquad (21)$$

which, after taking into account (13), leads to the differential equation

$$\frac{dT}{du} = A\frac{e^{-u}}{u}. \qquad (22)$$

The general solution of this equation has the form [3]

$$T(u) = \int_0^u \frac{Ae^{-x}}{x}dx + B, \qquad (23)$$

where A, B are constants. Taking into account the initial condition $T = T_0$ for $t = 0$ and boundary conditions

$T \to T_0$ for $r \to \infty, t \geq 0$,

and

$q = -2\pi r\lambda\frac{\partial T}{\partial r}$ for $r \to 0$, $t > 0$,

we can observe, with equation (4), that $u \to \infty$ for $r \to \infty$ or $t \to 0$ and that $T = T_0$. Then

$$T_0 = A\int_0^\infty \frac{e^{-x}}{x}dx + B \qquad (24)$$

Moreover, taking into account (4) and (5) to give

$$\frac{\partial T}{\partial r} = \frac{2u}{r}\frac{\partial T}{\partial u}, \qquad (25)$$

which after using (22) leads to the equality

$$\frac{\partial T}{\partial r} = A\frac{2u}{r}\frac{e^{-u}}{u} = 2A\frac{e^{-u}}{r}, \qquad (26)$$

it follows that

$$-\frac{q}{2\pi\lambda} = 2Ae^{-u}, \qquad (27)$$

which, after taking into account the boundary conditions, leads to the following:

$$A = -\frac{q}{4\pi\lambda}. \qquad (28)$$

With this equality, (24) takes the form [3]

$$T_0 = -\frac{q}{4\pi\lambda}\int_0^\infty \frac{e^{-x}}{x}dx + B, \qquad (29)$$

That is, it can be concluded that

$$B = T_0 + \frac{q}{4\pi\lambda}\int_0^\infty \frac{e^{-x}}{x}dx. \qquad (30)$$

Substituting into (23) yields

$$T(u) = -\frac{q}{4\pi\lambda}\int_0^u \frac{e^{-x}}{x}dx + T_0 + \frac{q}{4\pi\lambda}\int_0^\infty \frac{e^{-x}}{x}dx =$$

$$= \frac{q}{4\pi\lambda}\left[\int_0^\infty \frac{e^{-x}}{x}dx - \int_0^u \frac{e^{-x}}{x}dx\right] + T_0 = T_0 + \frac{q}{4\pi\lambda}\int_u^\infty \frac{e^{-x}}{x}dx. \quad (31)$$

Substitution of equation (4) into (31) yields:

$$T(r,t) = T_0 + \frac{q}{4\pi\lambda}\int_{\frac{r^2}{4\alpha\lambda}}^\infty \frac{e^{-x}}{x}dx. \quad (32)$$

Equation (32) is a basic equation, which is based on the interpretation of the thermal response test for the borehole heat exchangers [3]. Using an approximate solution of the integral in the equation (32) leads to the following relationship:

$$T(r,t) \cong T_0 + \frac{q}{4\pi\lambda}\left[ln\left(\frac{4\alpha t}{r^2}\right) - \gamma\right]. \quad (33)$$

A slope calculation results in measurements of the distance *k* in $T = f(\ln(t))$ so as to establish an effective thermal conductivity of the BHE using:

$$\lambda_{ef} = \frac{Q}{4\cdot\pi\cdot H\cdot k} = \frac{q}{4\cdot\pi\cdot k} \quad (34)$$

3. Numerical model

The governing energy equations for three-dimensional unsteady heat transfer in the geothermal system are solved for the soil (domain 1) and in the borehole heat exchangers (domains 2) [4, 5]. The governing energy balance equation for three-dimensional unsteady heat transport in the geothermal system takes the form for the soil:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right) + s \quad (35)$$

where ρ, c_p, and λ denote respectively the soil density, specific heat at constant pressure and thermal conductivity. The source term *s* allows other effects in the soil like underground water flow, natural heat sources and/or phase changes to be taken into consideration.

Assuming there is no chemical reaction or phase change in the working fluid, an energy balance for the first sub-domain (borehole heat exchangers) can be written as:

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \mathbf{v}\cdot\nabla T = \nabla\cdot(\lambda_i \nabla T) + s \quad (36)$$

Using the mathematical model and numerical procedures [6], we study system performance, which depends on the soil properties, the heat exchanger types (coaxial, single U-tube and double U-tube) and total power. The temperature of the surface varies over time according to average weather conditions. The heat exchanger tubes (height *H* = 78 m) are filled with working fluid (30% water solution of glycol) and in thermal equilibrium with the soil.

The following material properties are set for the pipe: density of 912 kg/m^3, specific heat at constant pressure of 1200 J/(kg·K) and thermal conductivity of 0.45 W/(m·K) [7]. The space between the outer tube and soil is fully filled with cement with a thermal conductivity of k = 1.0, 1.5, 2.0 and 2.5 W/(m·K). The working fluid flow rate is q = 20 l/min.

A control volume method is used on a cartesian grid to solve model equations (35)-(36) and the Adams-Bashforth method is used to approximate the unsteady terms. Either central difference CDS or a hybrid method is used for the convective/diffusive terms. The time step is not constant during the computations but varies so that it is optimal at any given time. The grid is developed using a *local grid refinement* technique with an iterative convergence procedure. Validation is performed for several cases. For the test case with unsteady infinite linear source the temperatures differ by less than 0.1% from the analytical solutions.

Variability in the subsurface thermal conductivity resulting from the differing lithologies of the formations has little effect on the temperature of the heat carrier exiting the heat exchanger T_{out}. The weighted average of the conductivity (which involves the specific heat and the density) of the entire rock mass is adequately representative for most cases and can be applied to a wide range of heat carriers. For example, an analysis the impact of groundwater filtration on the temperature of the heat carrier in a borehole heat exchanger [9], where the aquifer is about 20% of the length of the exchanger, showed that the value of the water flow rate of about 20 m·year^{-1} does not significantly affect the heat carrier temperature. However, increasing the rate of groundwater movement to 200 m·year^{-1} results in a noticeable change in the bottom temperature for a low conductivity rock [9]. For completeness in an analysis, it is prudent to consider the use of sealing materials, depending on the geological conditions (e.g., rock thermal conductivity).

4. Results and discussion

The numerically evaluated thermal response tests are carried out with the following parameters:

- heating power: 4 kW,
- heat carrier flow rate: 20 dm^3 min^{-1}, and
- time of heating: 100 h.

Several results and findings are presented, including:

- the temporal variation of the heat carrier temperature exiting the BHE for various BHE construction types and grout with a thermal conductivity of 2.5 W·m^{-1}·K^{-1} (Figs. 1 and 2),
- the variation of the effective thermal conductivity of the borehole heat exchanger with grout thermal conductivity and type of heat exchanger (Table 1), as well as the thermal resistivity defined as inverse of conductivity, and
- the variation of effective thermal conductivity of the borehole heat exchanger with grout thermal conductivity and heat exchanger type (Fig. 3).

Figure 2 shows the temperature dependence of the logarithm of time for the three BHEs and a linear regression for each BHE and the corresponding straight line equation. The dependence of the effective thermal resistivity R_{eff} on the thermal conductivity of grout is shown in Figure 4. For comparison, the dependence of the borehole thermal resistivity R_b on thermal conductivity of grout is shown in Fig. 5 for three cases of U-tube BHEs. The curves exhibit similar variations, but values of resistance are different because $R_{eff} \neq R_b$ [10].

5. Conclusions

The thermal conductivity of the grout in a borehole influences the effective thermal conductivity and resistivity of BHE, especially when the value of conductivity of grout is low. Using numerical simulations, for three kinds of BHE construction (coaxial, single U-tube and double U-tube), it is shown that the most advantageous values of thermal parameters (λ_{eff}) affects the construction of double U-tubes. Normally the preferred construction is a coaxial BHE. The calculations assume BHE dimensions like those of the system in the Geoenergetics Laboratory of the Drilling, Oil and Gas Faculty in AGH University of Science and Technology in Krakow, Poland (i.e., a coaxial BHE with a diameter of external pipe of 60 mm, and internal pipe of 40 mm). The lowest effective thermal conductivity is observed with TRTs based on existing laboratory BHE.

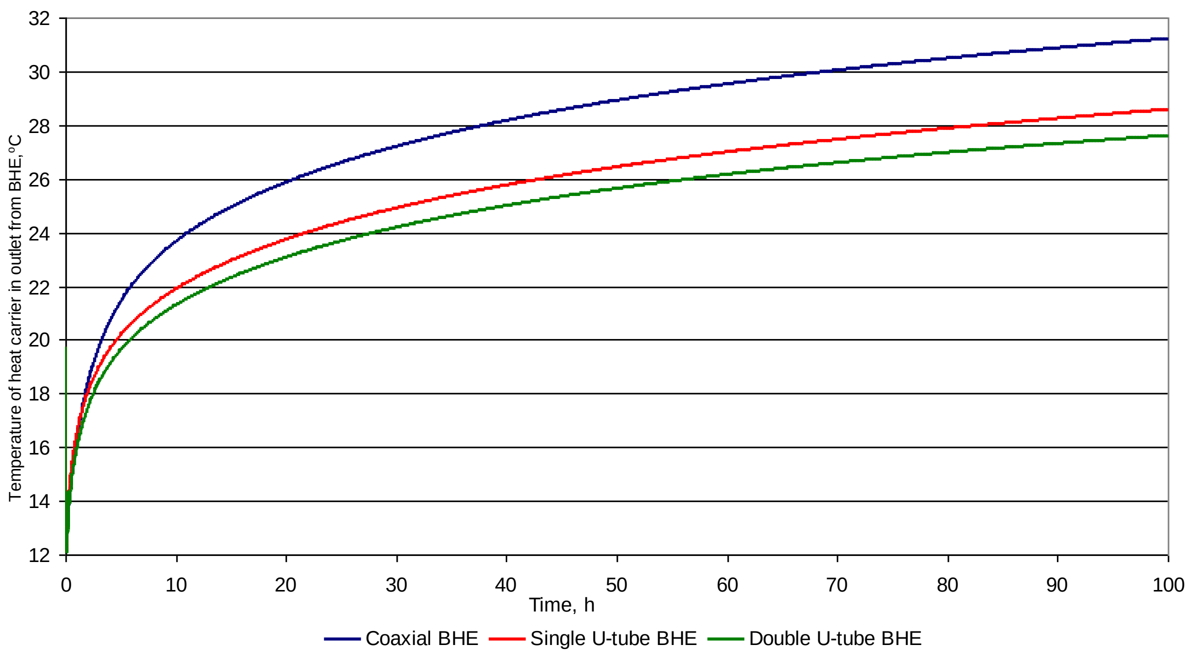

Figure 1. Temperature dependence of the heat carrier exiting the BHE with time, for grout with a thermal conductivity of 2.5 $W{\cdot}m^{-1}{\cdot}K^{-1}$ and for various BHE construction types.

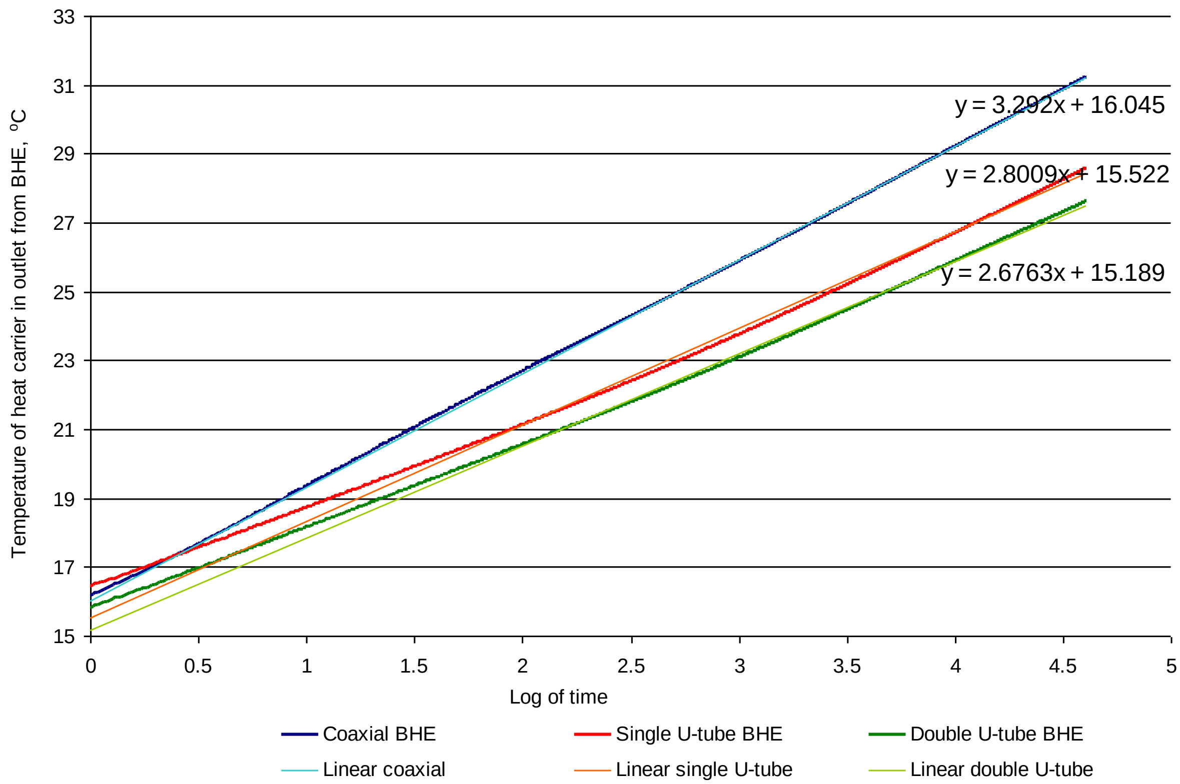

Figure 2. Temperature dependence of the heat carrier exiting the BHE with logarithm of time, for grout with a thermal conductivity of 2.5 $W{\cdot}m^{-1}{\cdot}K^{-1}$ and for various BHE construction types.

Table 1. Dependence of effective thermal conductivity of the borehole heat exchanger on grout thermal conductivity of and heat exchanger type.

BHE type	**Thermal conductivity of grout ($W{\cdot}m^{-1}{\cdot}K^{-1}$)**			
	1.0	**1.5**	**2.0**	**2.5**
Effective thermal conductivity of BHE, λ_{eff}, $W{\cdot}m^{-1}{\cdot}K^{-1}$				
Coaxial	1.071	1.163	1.211	1.24
Single U-tube	1.338	1.404	1.438	1.46
Double U-tube	1.409	1.473	1.505	1.525
Effective thermal resistivity of BHE, $R_{eff}=\lambda_{eff}^{-1}$, $m{\cdot}K{\cdot}W^{-1}$				
Coaxial	0.934	0.860	0.826	0.806
Single U-tube	0.747	0.712	0.695	0.685
Double U-tube	0.710	0.679	0.664	0.656

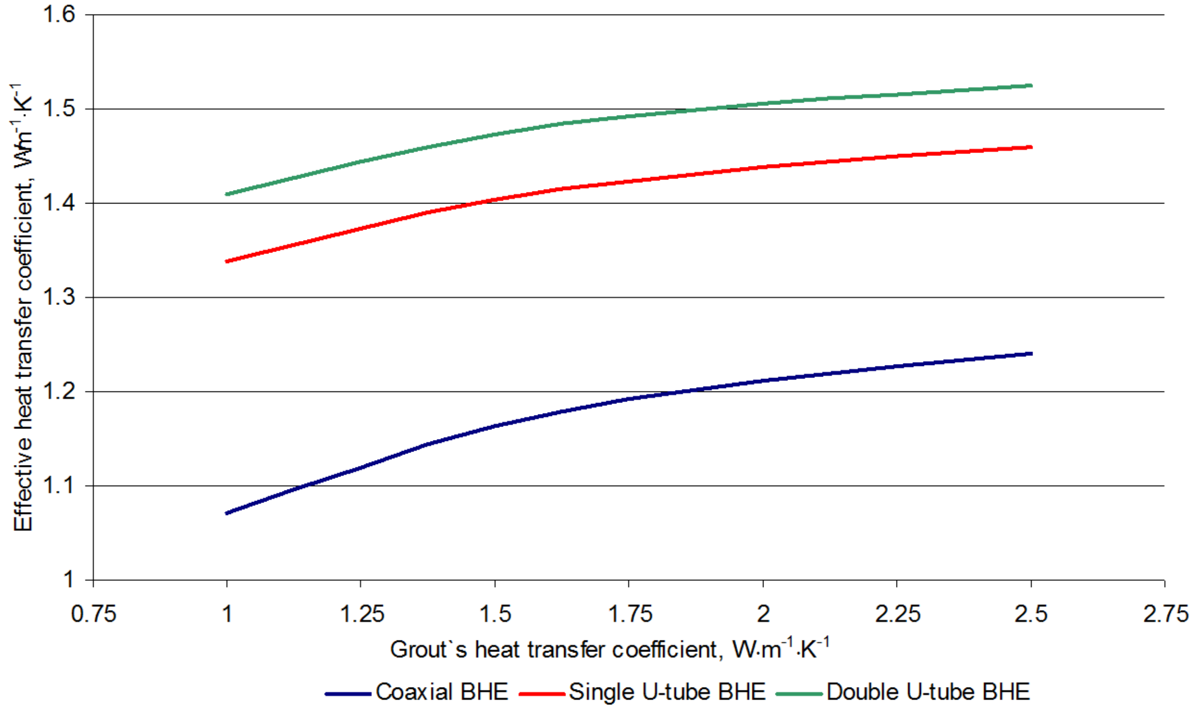

Figure 3. Dependence of effective thermal conductivity of the borehole heat exchanger on grout thermal conductivity and type of borehole heat exchanger.

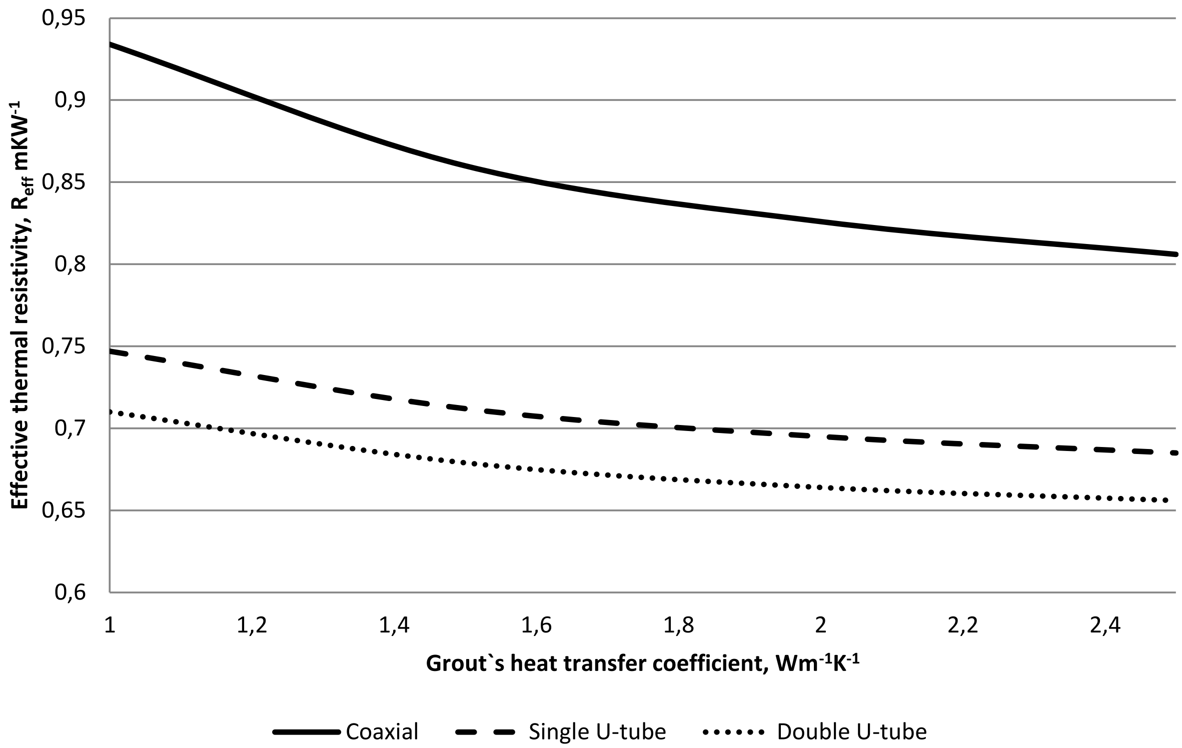

Figure 4. Dependence of effective thermal resistivity of the borehole heat exchanger on grout thermal conductivity and type of borehole heat exchanger.

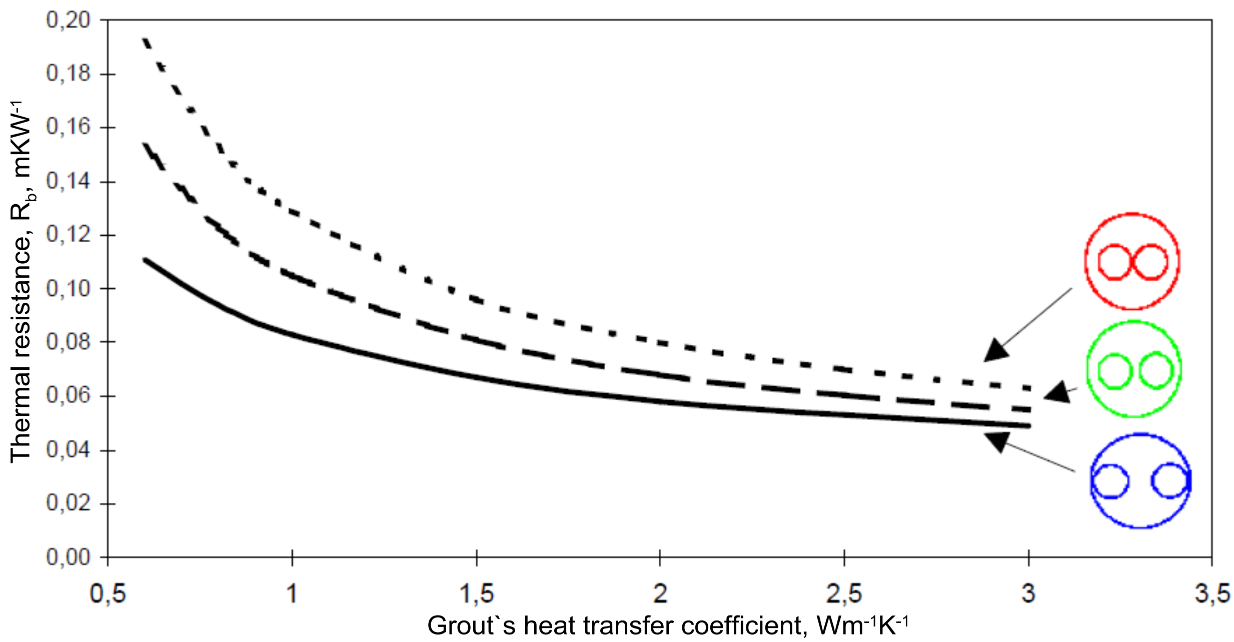

Figure 5. Dependence of effective thermal resistivity of the single U-pipe borehole heat exchanger on grout thermal conductivity and distance of pipes [8].

6. Nomenclature

C_p	specific heat
H	depth of BHE
q	unit thermal power
Q	thermal power
r	radius
t	time
T	temperature
λ	thermal conductivity
ρ	density
γ	Euler constant

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